Report on the thesis "Convex sets, barycentric algebras and beyond"

One of the most important classes of binary modes are convex sets with the weighted mean operations. As algebras, convex sets form a quasi-variety and the variety they generate is called the barycentric algebras.

One of the questions studied in this thesis is the equational basis of barycentric algebras. A standard basis consists of the idempotency, the skew-commutativity and the skew-associativity. A long standing question was whether the skew associativity could be replaced by the entropicity. The question was answered negatively in this thesis by constructing a class of so called threshold-t barycentric algebras. These algebras are idempotent, skew commutative and entropic but not skew associative.

When we have a new class of algebras that generalizes a well-established class, a natural question is: "What properties remain in the broader class?" The author was able to show that many proofs for barycentric algebras apply for threshold-t barycentric algebras as well. Unfortunately, there is a mistake in the proof of Corollary 2.4.9 but I believe that the result is correct nevertheless.

In the sequel, the author introduces some generalizations of the variety of threshold-t barycentric algebras. Again, many properties of barycentric algebras are preserved. For instance, in the last chapter the author replaces the field $\mathbb{R}$ by an arbitrary ring; the major issue is then how to define the notion of an interval so that we can have convex sets.

The results of the thesis are not very deep; actually the only deeper idea are the threshold-t barycentric algebras themselves as a counterexample to Keimel's problem. The other results are fairly straightforward. On the other hand, "straightforward" does not mean easy and the author proved that he mastered the mathematical craft. Therefore I find that the thesis is sufficient to grant a PhD and recommend the award of a PhD degree to Adam Komorowski.

A detail worth noting is the choice of references: most of the cited articles are decades old. Moreover, the only articles from this century are articles by prof. Romanowska. This phenomenon should be definitely explained during the defence of the thesis.

Major issues

- p.13: The "definition" of an Ω-semilattice is unclear. Let us have a support set A and some operations Ω. Do we say that (A, Ω) is an

Ω-semilattice if there exists a binary idempotent commutative associative operation $\cdot$ on A such that, for every $\omega \in \Omega$, $x_1x_2 \dots x_n\omega = x_1 \cdot x_2 \cdots x_n$?
Or the quantifiers are the other way round, saying, for every $\omega \in \Omega$, there exists a binary idempotent commutative associative operation $\cdot$ such that $x_1x_2 \dots x_n\omega = x_1 \cdot x_2 \cdots x_n$?

- p.27: Theorem 2.3.5 is not correctly formulated since the notion of an "algebra generated by some elements" is not defined. A correct version is: "... threshold-t barycentric subalgebra of $(I, \underline{\underline{I^o}})$ generated by $\{0, 1\}$..."
- p.33: In the proof of Corollary 2.4.9 we cannot use Theorem 2.3.5 because we do not know whether A is a subalgebra of a convex set.
- p.34: Theorem 2.4.12 depends on Corollary 2.4.9, although it is not explicitly stated.
- p.43: In the proof of Theorem 3.1.13 you implicitly use Theorem 1.6.2. This reference should be explicit.
- p.45: In Definition 3.2.2 you probably mean the variety generated by all the algebras of type (A, Ω), where A is an affine space over $\mathbb{F}$ and $\Omega = \{\underline{\underline{p}};\ p \in \mathbb{F}\}$.
- p.63–65: The examples should be rather propositions if they require proofs. Moreover, J' and J'' depend on R' and therefore a more convenient notation would be $J'_{R'}$ and $J''_{R'}$ (and, of course, we drop the index if R' is clear from the context).
- p.67: Proposition 4.3.3 is ill-formulated: B is a subset of A, a reduct is an algebra. A set is not an algebra, unless we implicitly assume some operations on it. Here we have three different sets of operations, namely $\underline{R}$, $\underline{S}$ and $\underline{T}$ and none of them is implicit on B. A probable meaning is: "B is closed on the operations $\underline{S}$ precisely when B is closed on the operations $\underline{T}$.

Small mistakes and misprints

- p.9: The arity is defined with the infix notation and then used with the postfix notation.
- p.9: The third paragraph starts with small a.

- p.9: The clone is defined by the clone.
- p.15: Equation (1.6.5) is missing the right side.
- p.16: Missing full stop at the end of the page.
- p.37: Symbols s and t are both used in two different meanings.
- p.39: $w_c^s = v_c^s$.
- p.62: In example 4.2.11 replace "may not" by "need not".
- p.63: In the first line, two commas are missing and $a + b$ is not defined.
- p.65: The definition of $J''[k]$ should be $\{f(k) \mid f \in J''(x)\}$.
- p.73: Lemma 4.4.8 contains Jam-mison.
- p.78: References [27] and [28] are equal.
- p.78: In reference [44] we can see "frepresentation".

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